Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$c_0 = 1;\ c_1 = C_1 - m;\ c_2 = C_2 - mc_1 - \frac{m(m-1)}{1.2};$$

$$c_3 = C_3 - mc_2 - \frac{m(m-1)}{1.2}c_1 - \frac{m(m-1)(m-2)}{1\,.\,2\,.\,3};\ \text{\&c.},$$

and thus obtain

$$(1+z)^n = 1 + (C_1 - m)z + \left(C_2 - mc_1 - \frac{m(m-1)}{1.2}\right)z^2$$

$$+\left(C_3 - mc_2 - \frac{m(m-1)}{1.2}c_1 - \frac{m(m-1)(m-2)}{1\,.\,2\,.\,3}\right)z^3. \quad [5]$$

Since, by hypothesis, n may have any value, let $n = -m$, then [4] becomes $(1+z)^0 = 1+0+\ldots$, that is, $C_1 \ldots C_r$ each $= 0$; and substit'ng these values in [5], we obtain, after some simplifications:

$$(1+z)^{-m} = 1 + (-m)z + \frac{-m(-m-1)}{1.2}z^2 + \frac{-m(-m-1)(-m-2)}{1.2.3}z^3 + \ldots$$

where the exponent is any negative integer.

3. Again, since $(1+z)^{nm} = (1+z)^{n+(m-1)n}$, we can, if the expansion of $(1+z)^n$ is given, obtain that of $(1+z)^{nm}$ by substituting $(m-1)n$ for m in [2]; and, conversely, if the expansion of $(1+z)^{nm}$ be given, we can obtain that of $(1+z)^n$ by substituting $(m-1)n$ for m in [5].

Let $n = s \div m$, s and m being integers, then $(1+z)^{nm} = (1+z)^s$, and we have, by [3],

$C_1 = s,\ C_2 = \frac{s(s-1)}{1\,.\,2}\ldots.$ Substituting, in [5], these values for $C_1 \ldots C_r$, and $(m-1)(s \div m)$ for m, and simplifying, we obtain

$$(1+z)^{\frac{s}{m}} = 1 + \frac{s}{m}z + \frac{(s\div m)[(s\div m)-1]}{1\,.\,2}z^2 + \frac{(s\div m)[(s\div m)-1][(s\div m)-2]}{1\,.\,2\,.\,3}z^3 + \ldots\ldots$$

Hence, since s may be pos. or neg., we find, whatever n may be,

$$(1+z)^n = 1 + nz + \frac{n(n-1)}{1.2}z^2 + \frac{n(n-1)(n-2)}{1.2.3}z^3 + \ldots.$$

and thence, by putting $y \div x$ for z, and multiplying by x^n,

$$(x+y)^n = x^n + nx^{n-1}y + \ldots\ldots.$$

SOLUTION OF PROB. 315 BY CHAS. H. KUMMELL. (SEE P. 166.)—The total distance of A is $D_a = 1+2+3 = 6^{\text{in}}$. It is required to find the most probable similar quantity for B.

The most probable position of B's bull's eye is evidently the center of gravity of his shots, viz., that point for which

$$D_1^2 + D_2^2 + D_3^2 = (x-0)^2 + (y-0)^2 + (x-1)^2 + (y-1)^2 + (x-2)^2 + (y-3)^2 = \text{min'm}.$$

$$\therefore \qquad (x-0)+(x-1)+(x-2) = 0;\ x = 1.$$
$$(y-0)+(y-1)+(y-3) = 0;\ y = \tfrac{4}{3}.$$

We have then

$$D_1 = \sqrt{(1+\tfrac{16}{9})} = 1.667\ ,$$
$$D_2 = \sqrt{(0+\ \tfrac{1}{9})} = 0.333\ ,$$
$$D_3 = \sqrt{(1+\tfrac{25}{9})} = 1.944\ .$$

The total distance from the most probable center would be then

$$D_b = 3^{\text{in}}.944, < D_a.$$

It would not be fair however to compare D_b, which is only probable, with D_a, which is true.

The rule agreed upon for comparing the skill makes evidently an abstraction of the direction of the shot from the center; it only considers the dist. The question arises: what is (with this abstraction) the most probable distance of the center of gravity from the true center?

We have for the mean error ε of the center of gravity being the true center

$$\varepsilon = \sqrt{\frac{[D^2]}{3\times 2}} = \tfrac{1}{3}\sqrt{10} = 1.054 \text{ in.}$$

We have then the probability of the true center being in the circumference of a circle of radius r about the center of gravity

$$\theta(r) = \frac{dr}{\varepsilon_0\sqrt{2\pi}}\, e^{-\frac{r^2}{2\varepsilon_0^2}};$$

therefore the most probable distance r_0 of the center of gravity from the true center is,

$$r_0 = \frac{\int_0^\infty r\frac{dr}{\varepsilon_0\sqrt{2\pi}}e^{-\frac{r^2}{2\varepsilon_0^2}}}{\int_0^\infty \frac{dr}{\varepsilon_0\sqrt{2\pi}}e^{-\frac{r^2}{2{}_0^2}}} = \varepsilon_0\sqrt{\frac{2}{\pi}} = 0.841^{\text{in}}.$$

The most probable distance of each of the three shots from the true center is then $D'_1 = \sqrt{(r_0^2+D_1^2)} = 1.867$ in., $D'_2 = \sqrt{(r_0^2+D_2^2)} = 0.905$ in., $D'_3 = \sqrt{(r_0^2+D_3^2)} = 2.118$ in.; hence $D'_b = D'_1+D'_2+D'_3 = 4.890, < D_a$. B was therefore the most probable winner.

A more correct mode of comparison of their skill would be, however, by means of the measures of precision. We have the mean error for A;

$$\varepsilon_a = \sqrt{\left(\frac{1^2+2^2+3^2}{3}\right)} = \sqrt{\frac{14}{3}};\ \therefore h_a = \frac{1}{\varepsilon_a\sqrt{2}} = \tfrac{1}{2}\sqrt{\tfrac{3}{7}} = 0.327; \text{ for } B,$$

$$\varepsilon_b = \sqrt{\left(\frac{[D^2]}{3-1}\right)} = \sqrt{\frac{10}{3}};\ \therefore h_b = \frac{1}{\varepsilon_b\sqrt{2}} = \tfrac{1}{2}\sqrt{\tfrac{3}{5}} = 0.387.$$

However, the ratio of skill agrees satisfactorily from the two methods for,

$$D_a \div D_b = \tfrac{6}{4.890} = 1.23, \qquad \text{and } h_b \div h_a = \tfrac{0.387}{0.327} = 1.19.$$